

Impact of Covariance of Soil Moisture Sensor Measurements on Inverse Estimation of Soil Water Balance Parameters and on Soil Moisture Predictions

Marit G.A. Hendrickx^{*1,2}, Jan Vanderborght^{1,3}, Pieter Janssens⁴, Jan Diels^{1,2}

* marit.hendrickx@kuleuven.be

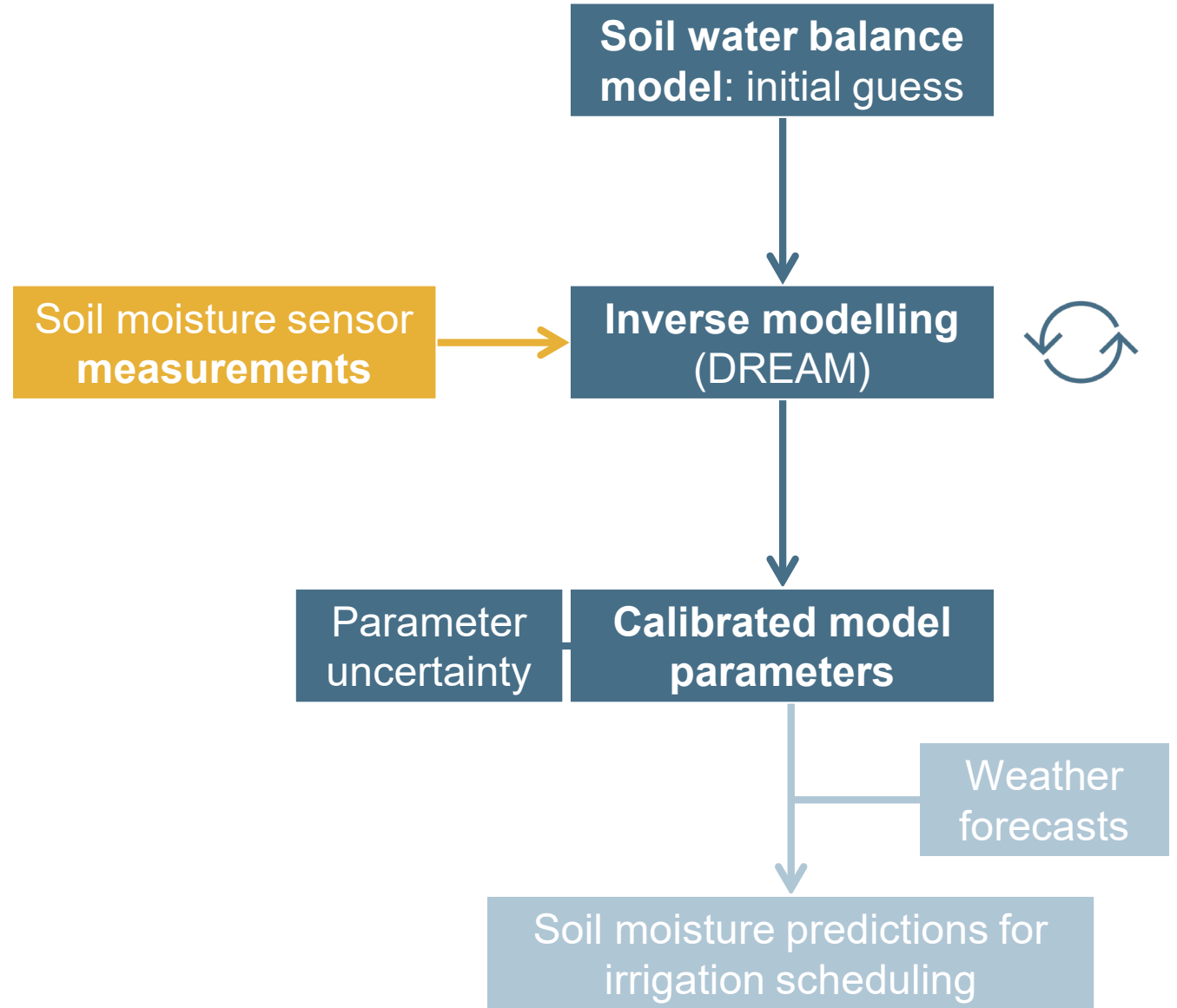
¹ Department of Earth and Environmental Sciences, KU Leuven, Celestijnenlaan 200E, 3001 Heverlee, Belgium

² KU Leuven Plant Institute (LPI), Kasteelpark Arenberg 31, 3001 Leuven, Belgium

³ Agrosphere Institute IBG-3, Forschungszentrum Jülich GmbH, 52425 Jülich, Germany

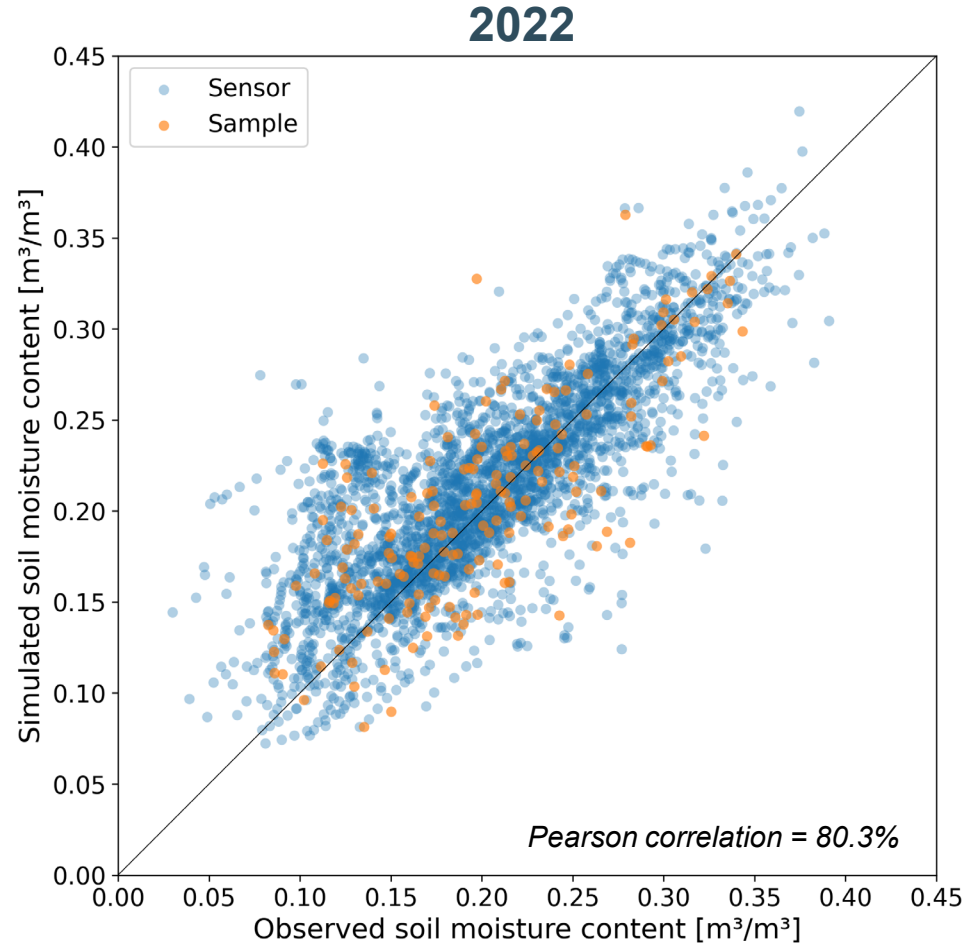
⁴ Soil Service of Belgium, Willem de Croylaan 48, 3001 Leuven, Belgium





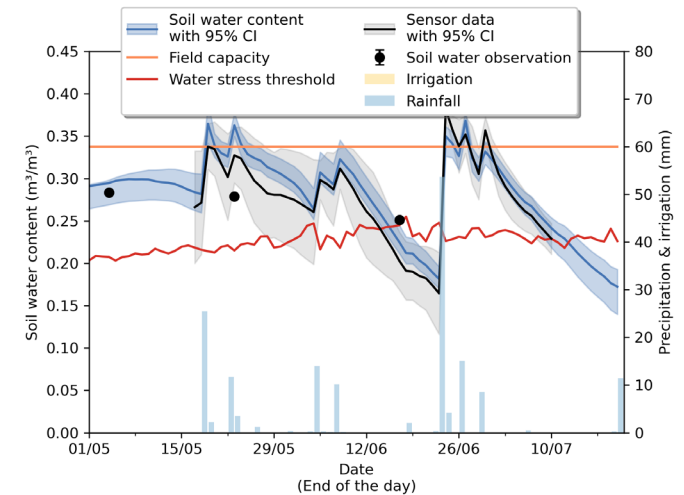
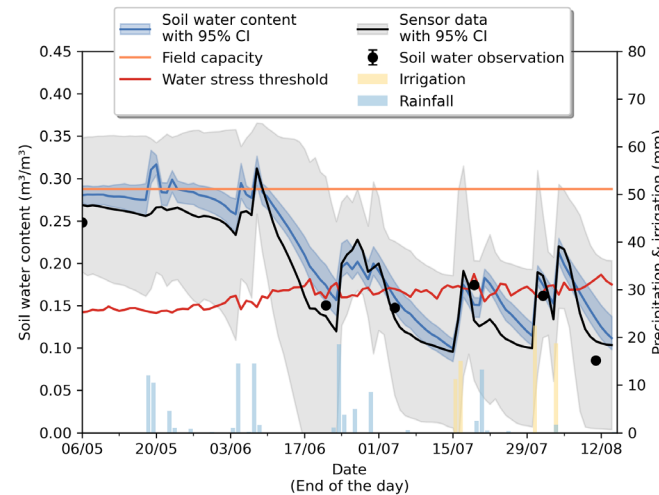
Introduction

Model predictions after inverse estimation



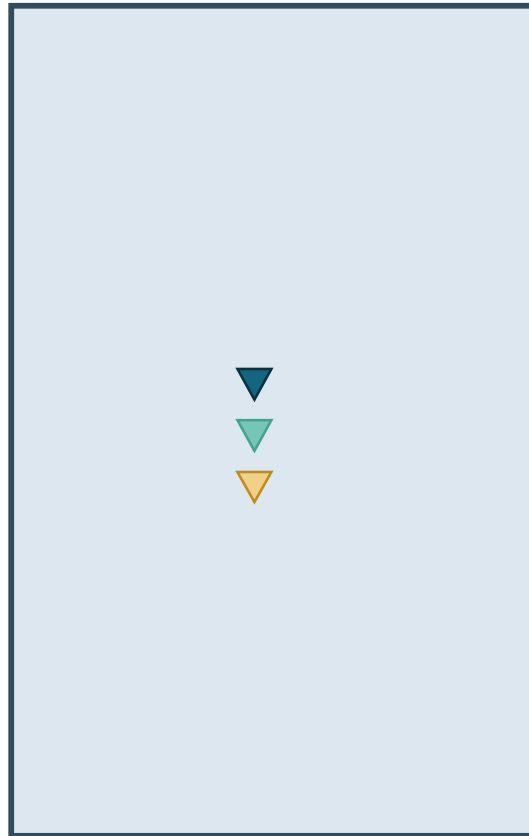
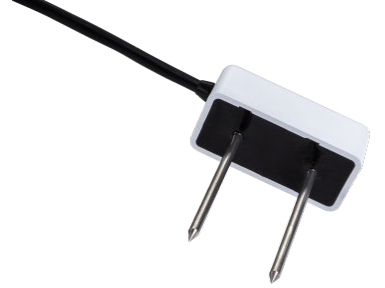
	Sensor data	Soil samples	Both
N [-]	3054	179	3233
ME [m^3/m^3]	-0.011	-0.005	-0.010
MAE [m^3/m^3]	0.029	0.031	0.029
RMSE [m^3/m^3]	0.040	0.040	0.040

Examples farmers fields 2022

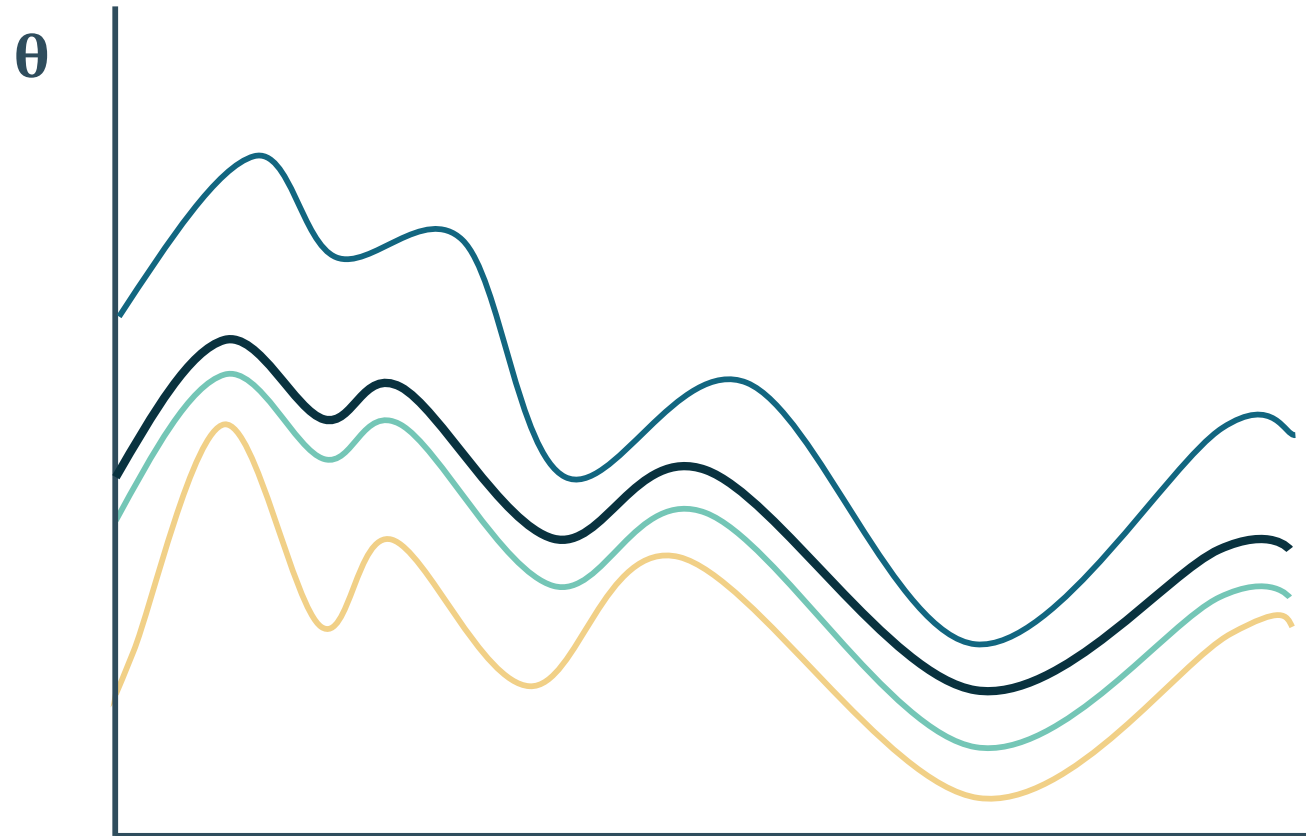


Introduction

Autocorrelated soil moisture sensor measurement errors



▽ Sensor



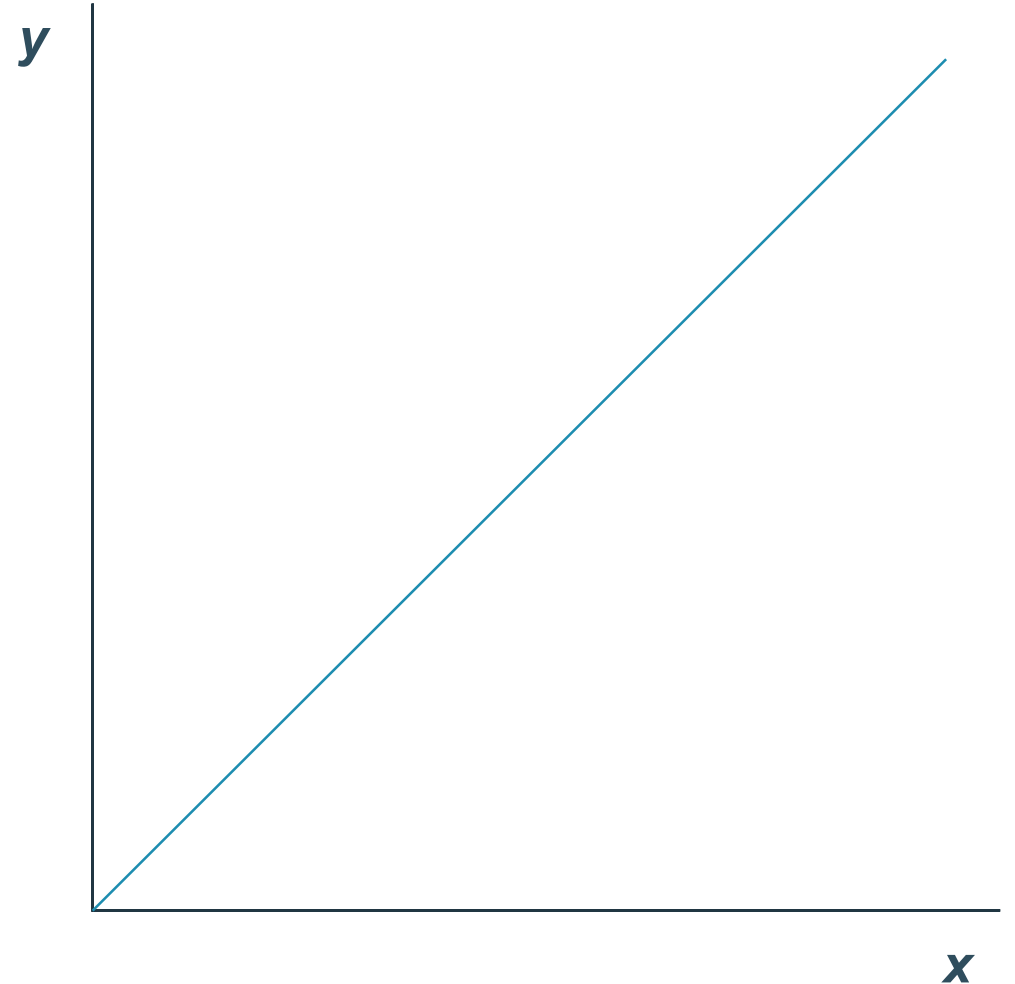
Location 1
Field average
Location 2
Location 3

time

Linear regression (GLS)

Intercept and slope uncertainty depending on measurement error correlation

$$y = a + bx$$

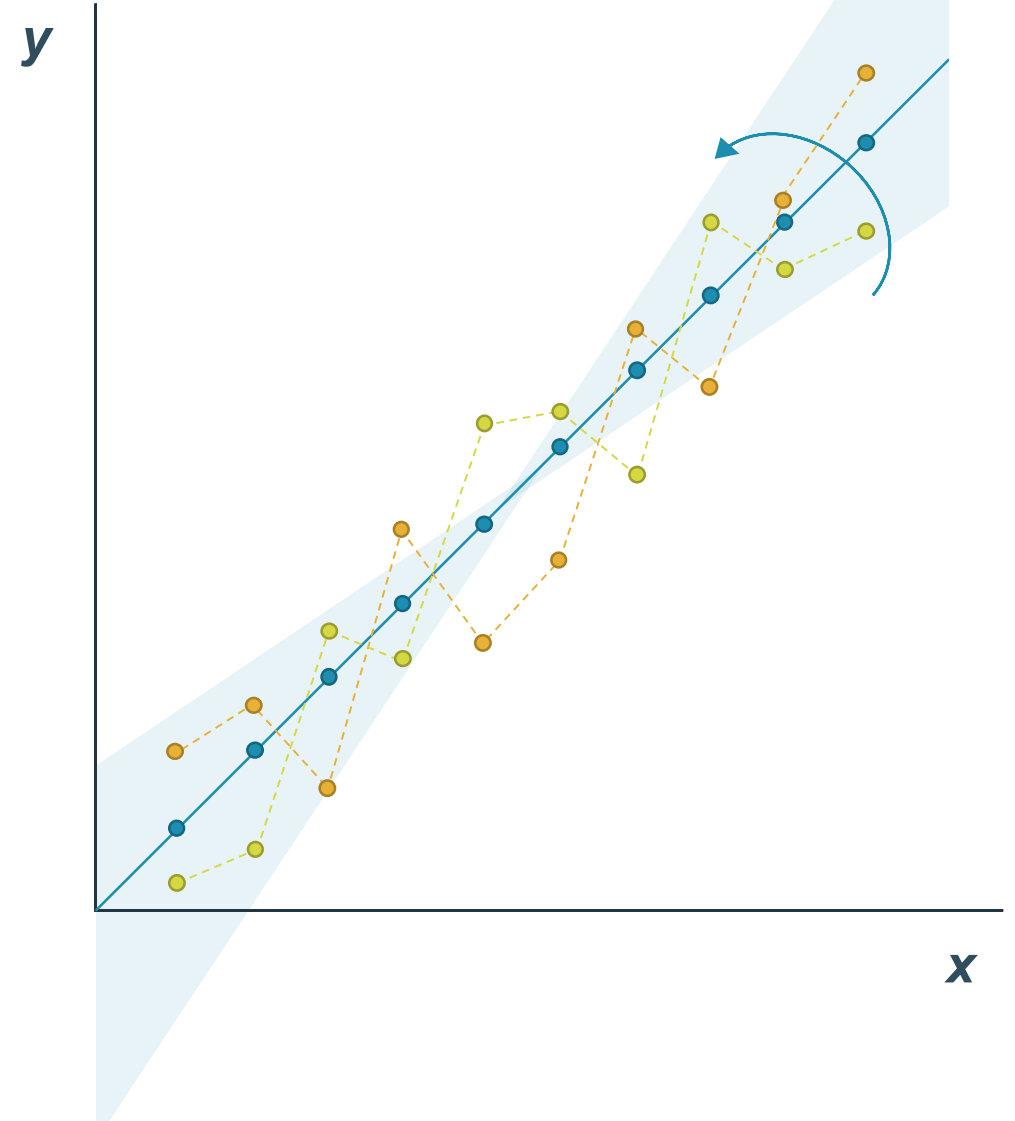


Linear regression (GLS)

Intercept and slope uncertainty depending on measurement error correlation

$$y = a + bx$$

Uncorrelated measurement errors



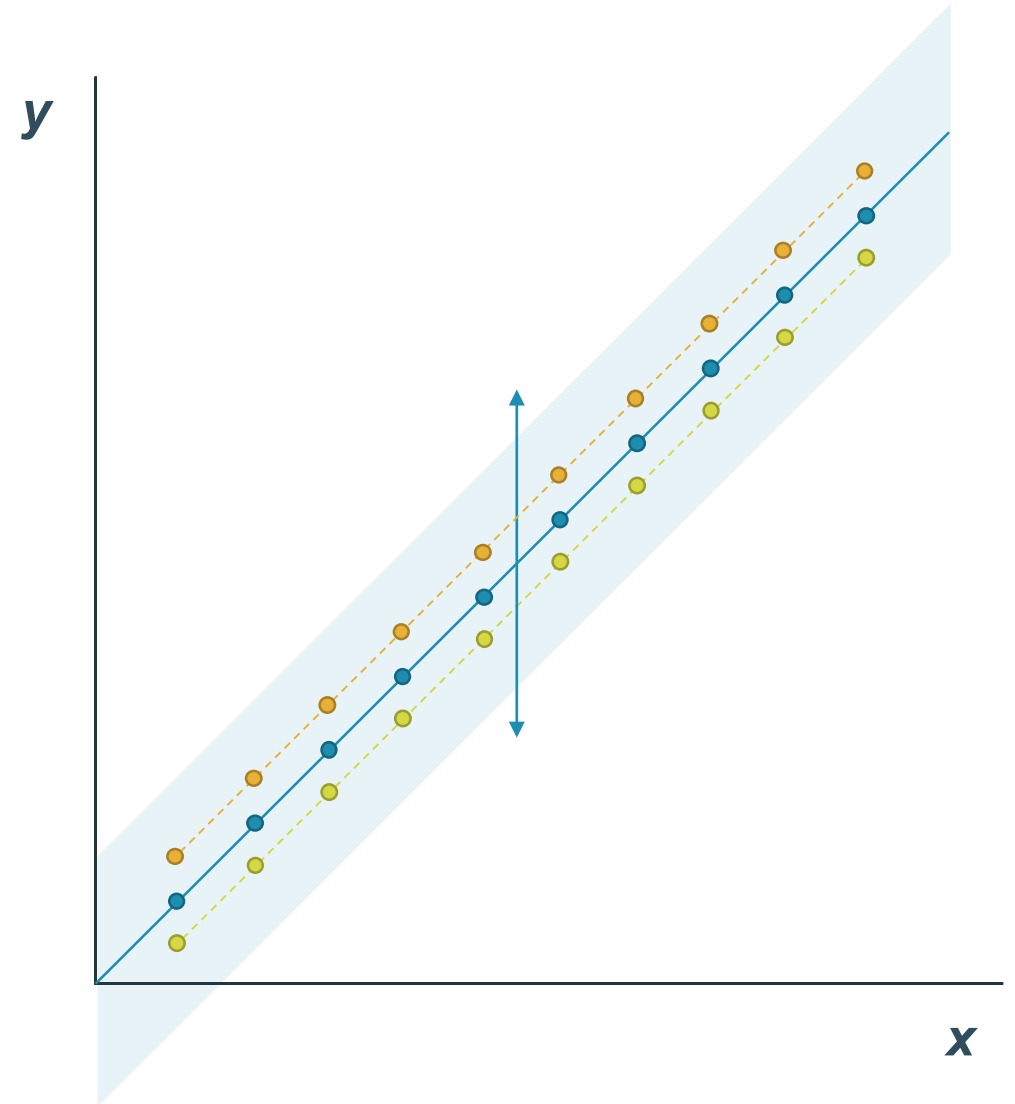
Linear regression (GLS)

Intercept and slope uncertainty depending on measurement error correlation

$$y = a + bx$$

$$COV_{GLS}(a, b) = \sigma^2(X^T \Sigma^{-1} X)^{-1}$$

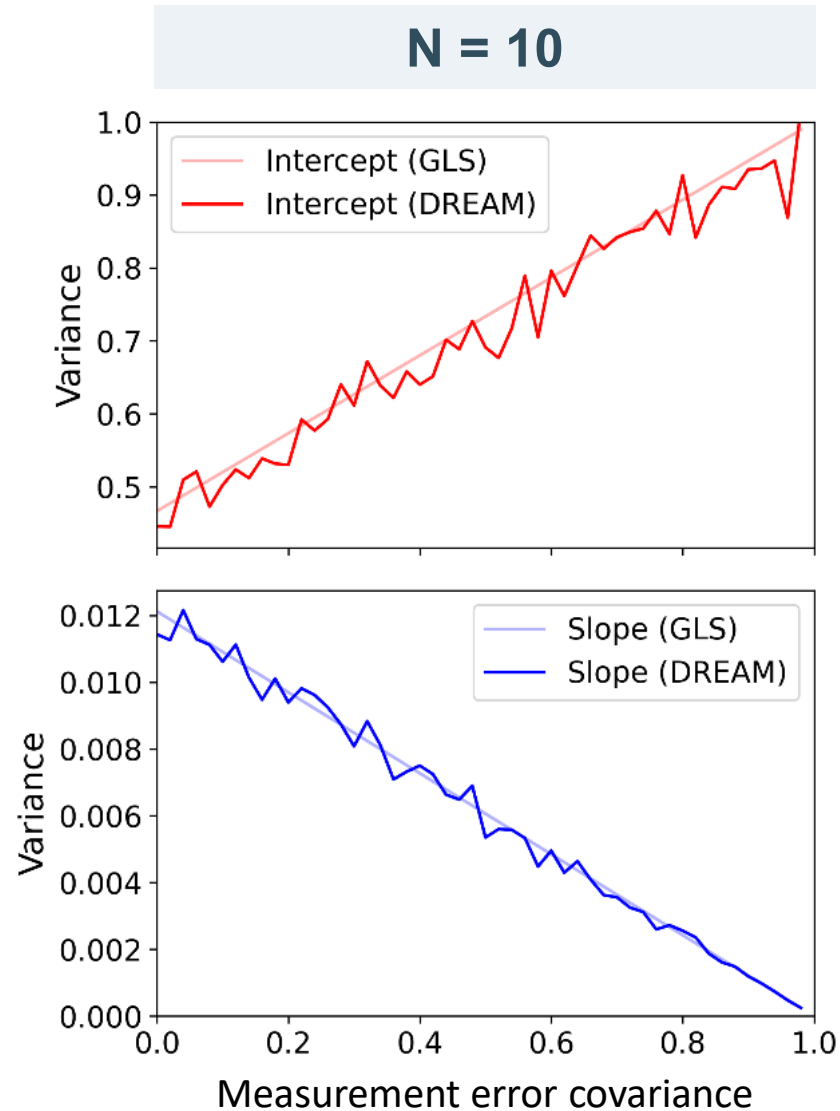
Perfect measurement error correlation



Linear regression (GLS)

Parameter uncertainty estimate & approximation with DREAM

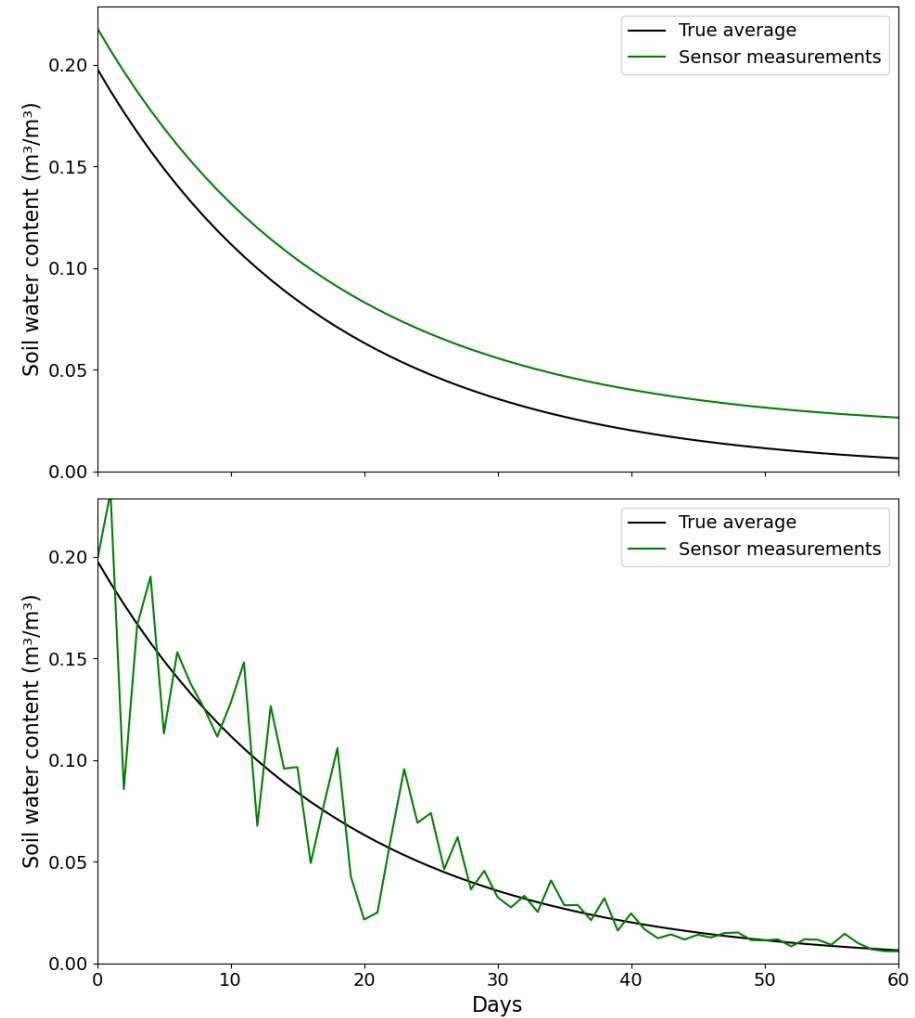
- Homoscedastic errors
- Constant covariance



Impact of ...

Soil moisture error covariance matrix

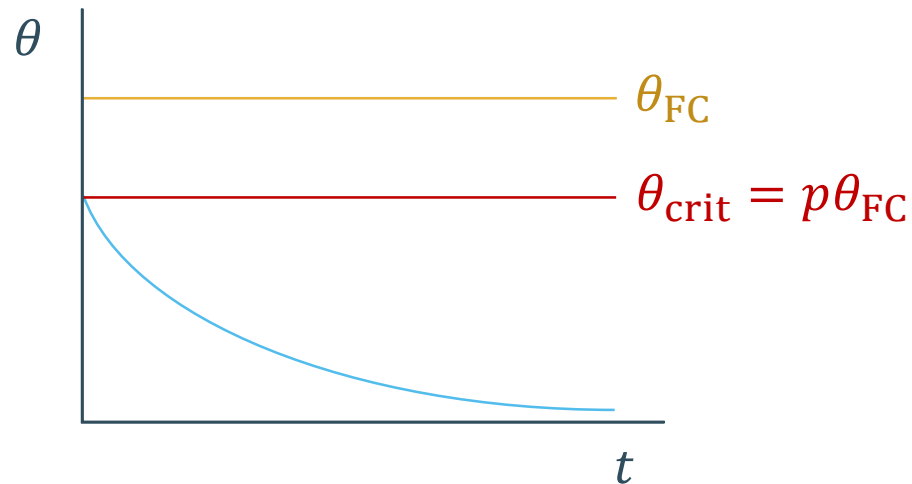
$$\begin{bmatrix} \text{Var}(x_1) & \dots & \text{Cov}(x_n, x_1) \\ \vdots & \cdot & \vdots \\ \text{Cov}(x_n, x_1) & \dots & \text{Var}(x_n) \end{bmatrix}$$



COVER

NO COVER

... on a simple soil water balance



Water balance

- Initial soil moisture $\theta_{ini} = \theta_{crit} = p\theta_{FC}$
- Constant ET_o
- Constant root depth Z
- No runoff, no capillary rise
- $\theta_{WP} = 0$

$$\frac{d\bar{S}}{dt} = -\alpha\bar{S}$$

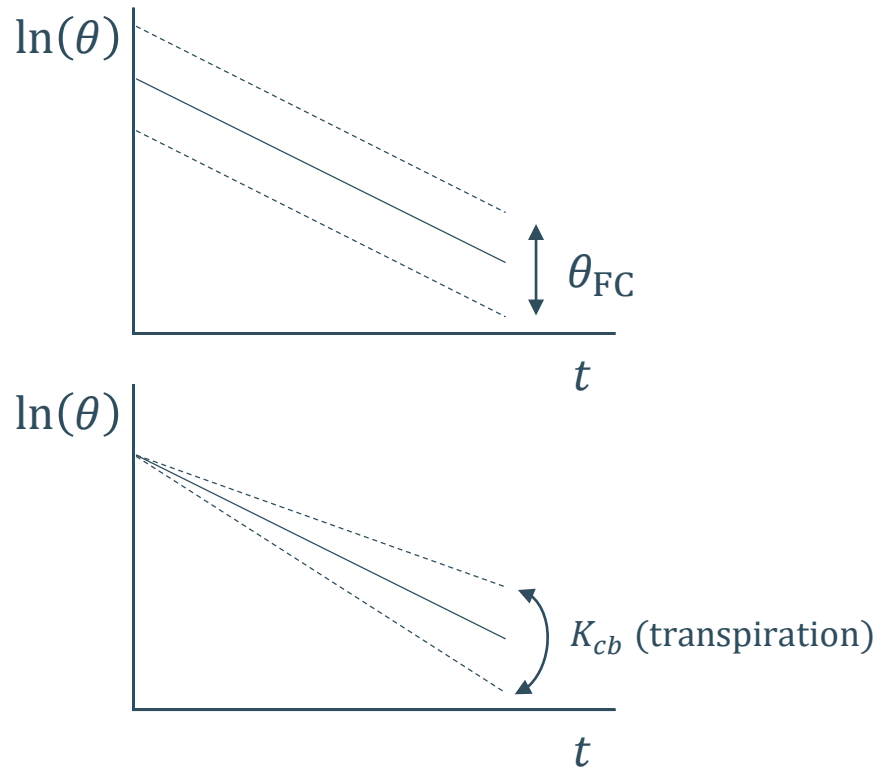
$$\alpha = \frac{K_{cb} ET_o}{Z}$$

$$\bar{S} = \frac{\bar{\theta}}{p\theta_{FC}} \quad (\text{for } \bar{\theta} < p\bar{\theta}_{FC})$$

$$\ln(\bar{\theta}) = -\alpha t + \ln(p\bar{\theta}_{FC}) \quad \text{or} \quad \bar{\theta} = e^{-\alpha t} p\bar{\theta}_{FC}$$

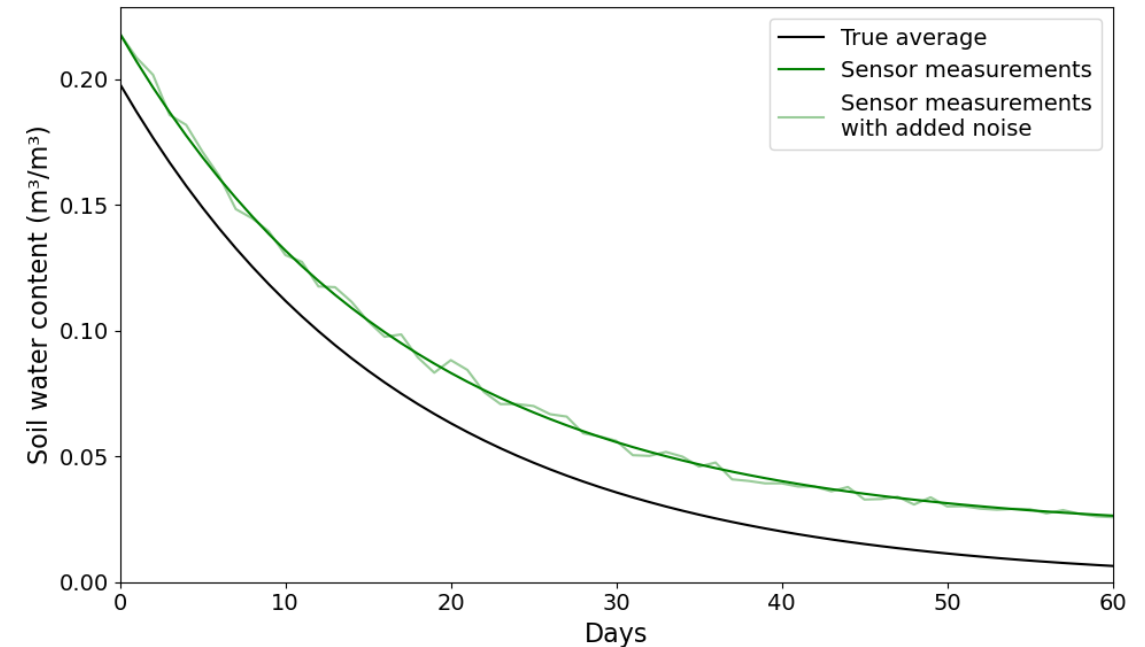
... on a simple soil water balance

$$\ln(\bar{\theta}) = -\alpha t + \ln(p\bar{\theta}_{FC})$$



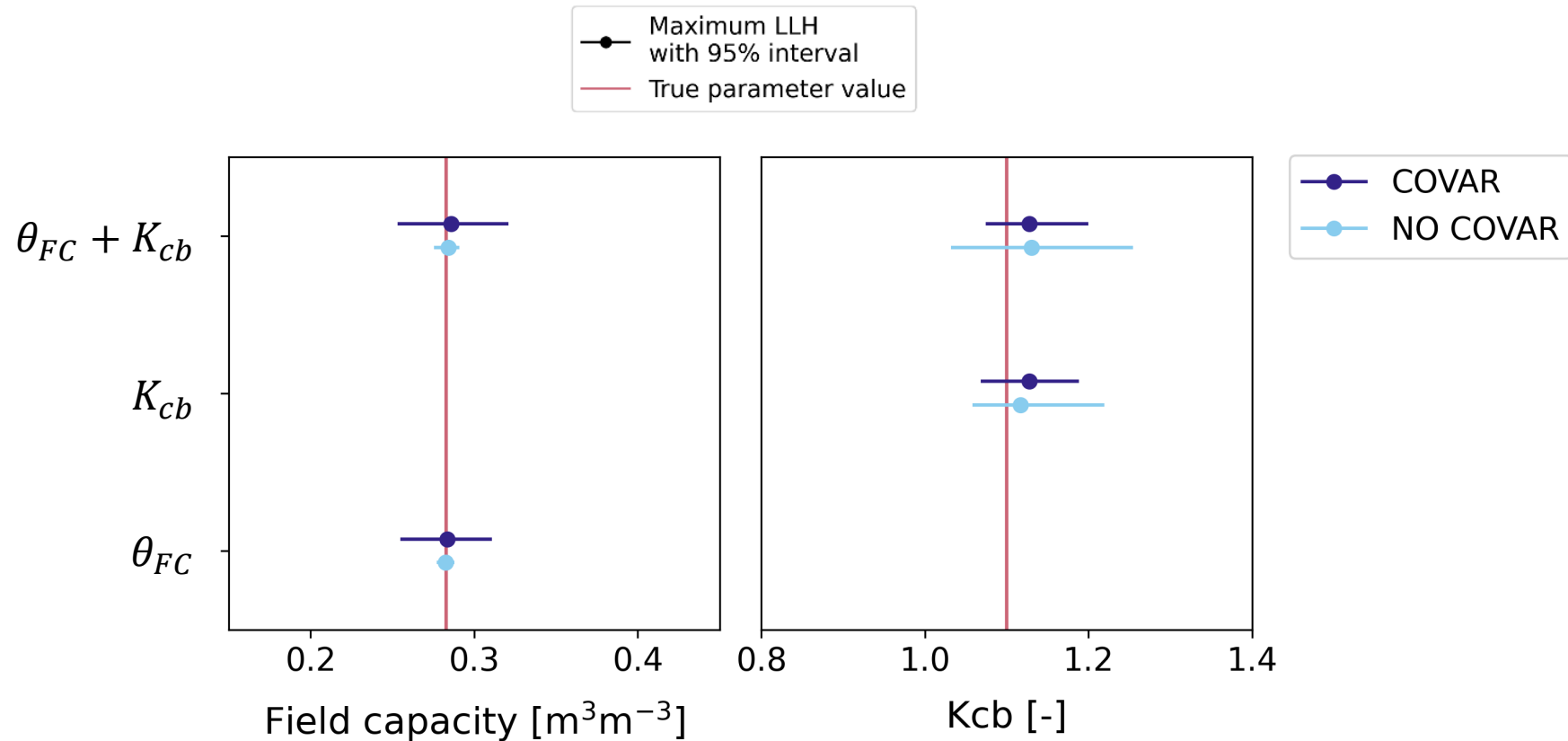
Uncertain parameters estimated in DREAM:

- Field capacity (θ_{FC})
- Crop coefficient (K_{cb})
- θ_{FC} & K_{cb}



With and without acknowledging error covariance

Parameter estimation • Soil moisture uncertainty • Parameter correlations

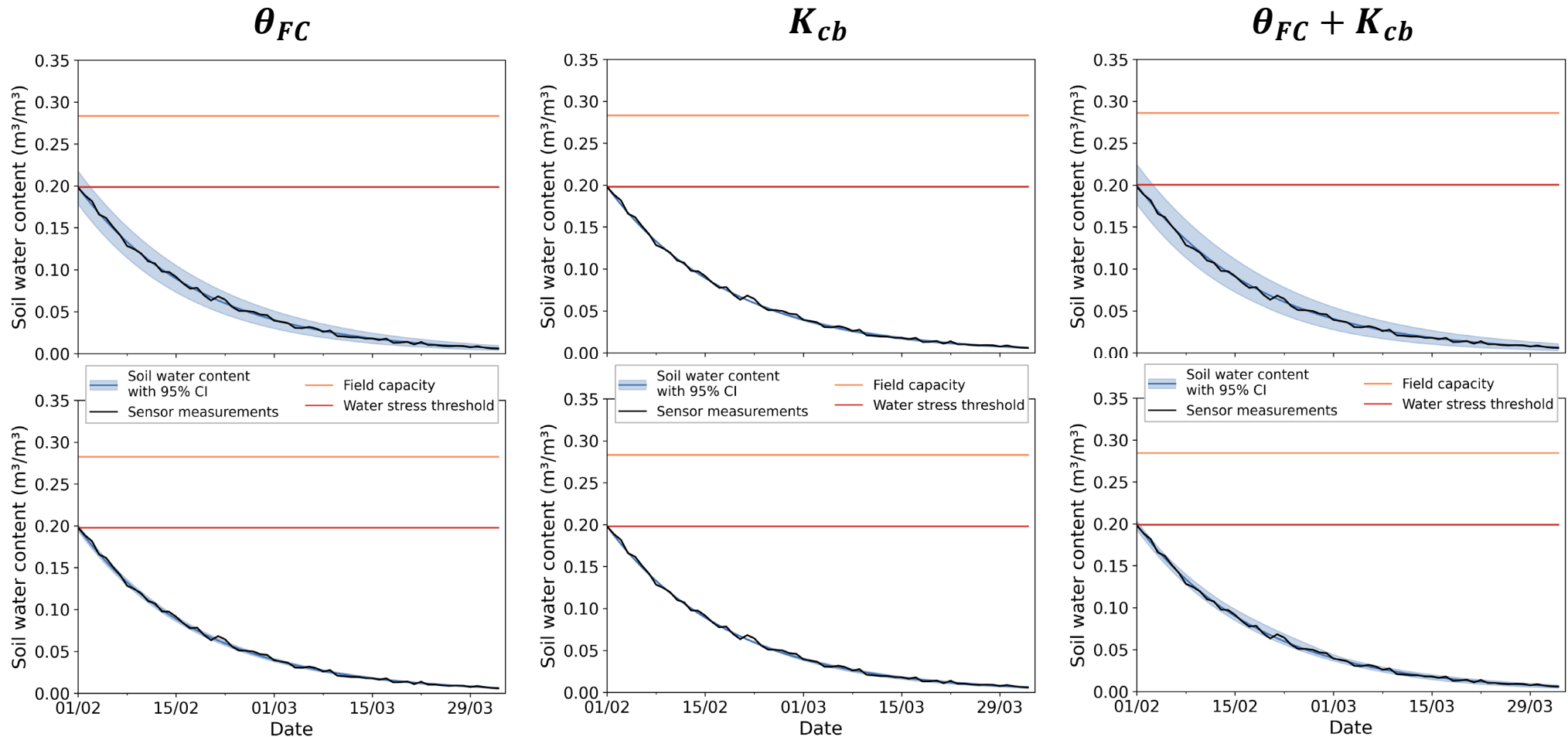


With and without acknowledging error covariance

Parameter estimation • Soil moisture uncertainty • Parameter correlations

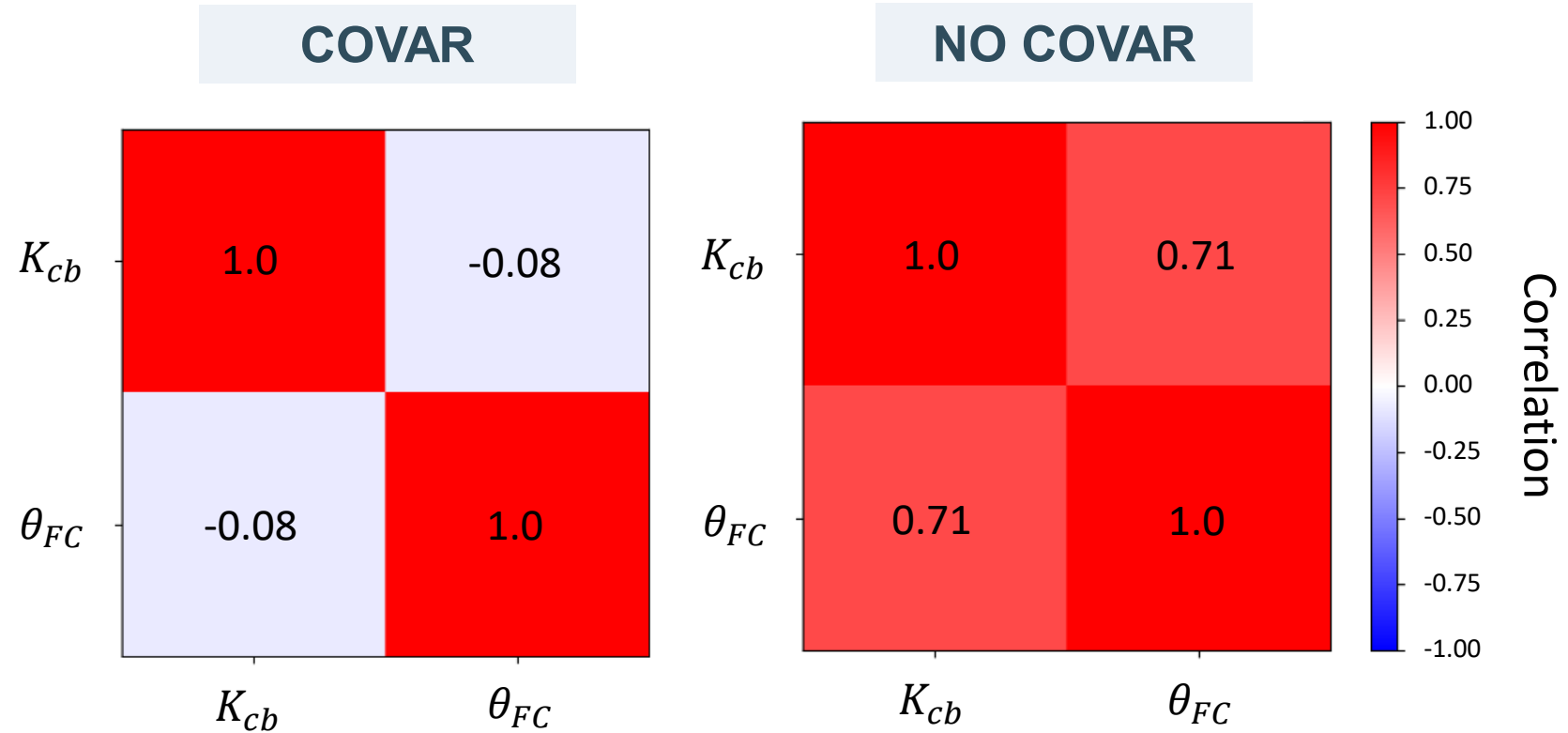
COVAR

NO COVAR



With and without acknowledging error covariance

Parameter estimation • Soil moisture uncertainty • Parameter correlations



Parameter covariance = $\sigma^2(X^T \Sigma^{-1} X)^{-1}$



Soil moisture sensor measurements

- Measurement errors (i.e. deviations from the field average) exhibit temporal correlation
- Neglecting measurement error autocorrelation is an incorrect assumption

Linear regression: Parameter estimation with GLS or DREAM

- No measurement error correlation
 - Low intercept uncertainty
 - High slope uncertainty
- Increasing measurement error correlation
 - Increasing intercept uncertainty
 - Decreasing slope uncertainty

Impact of measurement error autocorrelation on a water balance

- Impact on parameter estimation
 - Parameter specific: θ_{FC} acts like intercept, while K_{cb} acts like slope
- Impact on soil moisture uncertainty
- Impact on parameter correlations



marit.hendrickx@kuleuven.be

Soil and Water Management,
KU Leuven (Arenberg)
Celestijnenlaan 200e - box 2411
3001 Leuven

